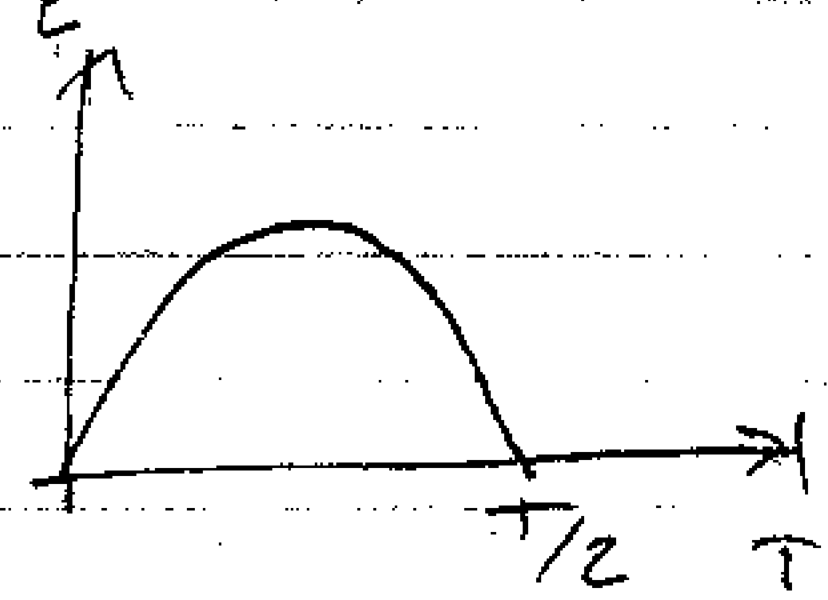
Prepa 2 ABR-JUL 2010 Johnny Rengifo

1) Para la señal de tensión de la figura halle:

a) Valor promedio

b) Valor eficaz

c) Factor de forma



$$E(t) = \begin{cases} A \operatorname{sen}(100t) & t \in [0, T/2] \\ \emptyset & \text{otro caso} \end{cases}$$

$$T = \frac{2\pi}{100}$$

$$E_{pro} = \frac{1}{T}\int_0^T E(t)\,dt = \frac{1}{T}\left(\int_0^{T/2} A\operatorname{sen}(100t)\,dt + \cancel{\int_{T/2}^{T} \emptyset\,dt}^{\,0}\right)$$

$$= \frac{A}{T}\left[-\frac{\cos(100t)}{100}\Bigg|_0^{T/2}\right] = \frac{A}{T}\left[\frac{-\cos\left(\cancel{100}\frac{\cancel{2}\pi}{\cancel{2}\cdot\cancel{100}}\right)}{100} + \frac{\cos(\emptyset)}{100}\right]$$

$$= \frac{A}{T}\left[\frac{1+1}{100}\right] = \frac{\cancel{100}A}{\cancel{2}\pi}\,\frac{\cancel{2}}{\cancel{100}} = \frac{A}{\pi}$$

$$E_{RMS} = \sqrt{\frac{1}{T}\int_0^T E^2(t)\,dt} = \sqrt{\frac{100A^2}{2\pi}\int_0^{T/2} \operatorname{sen}^2(100t)\,dt}$$

Recordar: $\operatorname{sen}^2 x = \dfrac{1-\cos(2x)}{2}$

$$E_{RMS} = 10A\sqrt{\frac{1}{2\pi}\int_0^{T/2}\frac{1-\cos(200t)}{2}\,dt} = 10A\sqrt{\frac{1}{2\pi}\left[\int_0^{T/2}\frac{dt}{2} + \int_0^{T/2}-\frac{\cos(200t)}{2}\,dt\right]}$$

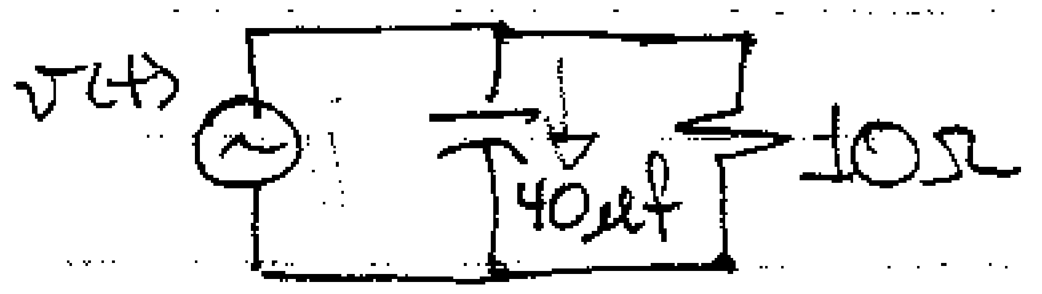
$$= 10A\sqrt{\frac{1}{2\pi}\left(\frac{t}{2}\Big|_0^{T/2} - \frac{\text{sen}(200t)}{400}\Big|_0^{T/2}\right)}$$

$$= 10A\sqrt{\frac{1}{2\pi}\left(\frac{T}{4} - \underbrace{\frac{\text{sen}\left(200\frac{2\pi}{100}\right)}{400}}_{0} + \underbrace{\frac{\text{sen}(0)}{400}}_{0}\right)}$$

$$= 10A\sqrt{\frac{1}{2\pi}\,\frac{2\pi}{(100)4}} = \frac{10\,A}{(10)\cdot(2)} = \frac{A}{2}$$

$$\text{Factor de Forma} = \frac{E_{RMS}}{E_{PROM}} = \frac{A/2}{A/\pi} = \frac{\pi}{2} = 1,5708$$

Para el circuito de la figura



$v(t) = \sqrt{2}\,V_{rms}\cos(\omega t)\ V$

$f = 50\,Hz$

$V_{rms} = 120\,V$

$$f = 50\,Hz \Rightarrow \omega = 314,1593\ rad/s$$
$$\Rightarrow T = 1/f = 20\,ms$$

a) Halle la corriente por cada elemento

b) Ecuación de la potencia instantánea que entrega la fuente

c) Valor efectivo de cada una de las corrientes

d) Función temporal de la energía que almacena el capacitor (Gráfico)

e) ¿Cuánta energía hay almacenada en el capacitor cuando $t = 5ms$ y $t = 20ms$?

f) Potencia promedio entregada por la fuente

Solución

$v(t) = v_C(t) = v_R(t) \Rightarrow$ Elementos en paralelo

$$i_C(t) = C \frac{dV_C(t)}{dt}$$

$$= C \frac{d(\sqrt{2}\, V_{rms} \cos(\omega t))}{dt} = -\sqrt{2}\, C\, V_{rms}\, \omega \operatorname{sen}(\omega t)$$

$$= \sqrt{2}\, V_{rms}\, \omega C \cos(\omega t + \pi/2) = \sqrt{2}\; 1{,}5080 \cos(\omega t + \pi/2)$$

Resistencia

$$i_R(t) = \frac{v(t)}{R} = \frac{\sqrt{2}\, V_{rms} \cos(\omega t)}{R} = \sqrt{2}\; 12 \cos(\omega t) \text{ A.}$$

$$i(t) = i_C(t) + i_R(t) = \sqrt{2}\; 1{,}5080 \cos(\omega t + \pi/2) + \sqrt{2}\; 12 \cos \omega t$$

Potencia instantánea

$p(t) = v(t)\, i(t) = \cancel{\ldots}$
$= \sqrt{2}\, V_{rms} \cos(\omega t) \left(\sqrt{2}\, 1{,}5080 \cos(\omega t + \frac{\pi}{2}) + \sqrt{2}\, 12 \cos \omega t \right)$

$p(t) = 511{,}8240 \cos \omega t \cos(\omega t + \frac{\pi}{2}) + 2880 \cos^2 \omega t \ W$

Valor efectivo de las corrientes ~~[illegible]~~

$I_C = 1{,}5080\ A \qquad I_R = 12\ A$

$$I_f = \sqrt{\frac{1}{T} \int_0^T \left(\sqrt{2}\, \underbrace{1{,}5080}_{I_C} \cos(\omega t + \frac{\pi}{2}) + \sqrt{2}\, \underbrace{12}_{I_R} \cos(\omega t) \right)^2 dt}$$

$$= \sqrt{\frac{1}{T} \left(2 (\underbrace{1{,}5080}_{I_C})^2 \cos^2(\omega t + \pi/2) + 4 I_R I_C \underbrace{\cos(\omega t + \pi/2) \cos \omega t}_{= -\operatorname{sen} \omega t \cos \omega t} + 2 (I_R)^2 \cos^2 \omega t \right.}$$

$$I_f = \sqrt{\frac{1}{T} \left(\int_0^T 2 I_C^2 \cos^2(\omega t + \frac{\pi}{2}) dt - \int_0^T 4 I_R I_C \operatorname{sen} \omega t \cos \omega t \, dt + \int_0^T 2 I_R^2 \cos^2 \omega t \right)}$$

$v = \operatorname{sen} \omega t$
$dv = \omega \cos \omega t$
$v_1 = \operatorname{sen}(T) = 0$
$v_0 = \operatorname{sen}(0) = 0$

$$I_f = \sqrt{\frac{1}{T} \left(2 I_C^2 \int_0^T \frac{1 + \cos(2\omega t + 2\pi/2)}{2} dt - \cancel{\frac{4 I_R I_C}{\omega} \int_{v_0}^{v_1} v \, dv} + \int_0^T 2 I_R^2 \left(\frac{1 + \cos 2\omega t}{2} \right) \right.}$$

$$I_f = \sqrt{\frac{1}{T}\left(I_C^2\left[t\Big|_0^T + \cancel{\frac{\operatorname{sen}(2\omega t+\pi)}{2\omega}\Big|_0^T}\right] + I_R^2\left[t\Big|_0^T + \cancel{\frac{\operatorname{sen}(2\omega t)}{2\omega}\Big|_0^T}\right]\right)}$$

$$\qquad\qquad\qquad\qquad 0 \qquad\qquad\qquad\qquad\qquad\qquad 0$$

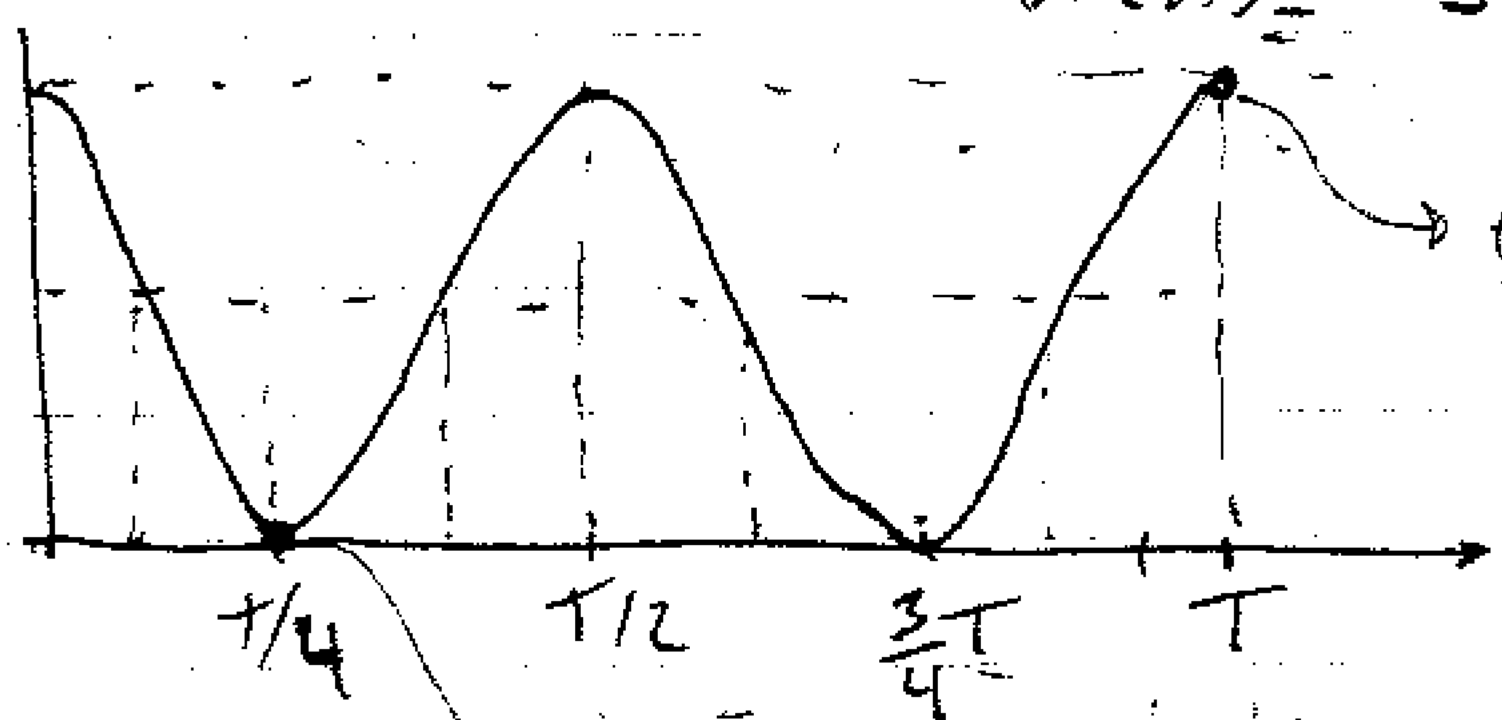
$$= \sqrt{\frac{1}{\cancel{T}}\left(\cancel{T} I_C^2 + \cancel{T} I_R^2\right)} = \sqrt{I_C^2 + I_R^2}$$

Energía en el Capacitor

$$W_C(t) = \frac{1}{2} C\, v^2(t) = \frac{1}{2}\sqrt{2}\, V\cos^2(\omega t) = 3{,}3941\cos^2(\omega t)\ \text{mJ}$$

Gráfica $\qquad W_C(t) = 3{,}3941\left(\frac{1+\cos(2\omega t)}{2}\right)$

$$W_C(t) = 1{,}6971 + 1{,}6971\cos 2\omega t \ \text{mJ}$$



$t = 20\text{ms} \Rightarrow W_C(t=20\text{ms}) = 3{,}3941\text{mJ}$

$t = 5\text{ms} \Rightarrow W_C(t=5\text{ms}) = 0\ \text{mJ}$